CHAPTER 4

ANGULAR DISTRIBUTION OF PHOTOPROTONS FROM $^{27}$Al

4.1 Introduction
The results of the previous chapter demonstrated well defined structure in the $^{27}$Al($\gamma,p$) and $^{27}$Al($\gamma,p$) cross sections. By inference from the strong population of the low-lying positive parity residual states of $^{26}$Mg, the $ld_{5/2}\rightarrow 1f_{7/2}$ single particle dipole transition should be the dominant 'doorway state' for the photonuclear reaction. To obtain further insight into the photoproton reaction mechanisms for $^{27}$Al proton spectra were measured at 7 angles centred about 90° in the laboratory frame using the apparatus described in Chapter 2.

The angular distribution of a photonuclear reaction provides information on the relative strengths of the various possible reaction channels defined by the multipolarity ($J^{\pi}$) of the intermediate state, the emitted particle's relative orbital angular momentum and the channel spin of the particle channel. It is thus possible that with a knowledge of the angular distributions the structure in the cross sections can be correlated with the strongest particle-hole configurations which contribute to the GDR of $^{27}$Al.

4.2 Experimental Details
A target was fashioned from a foil of the same thickness as used for the measurements reported in Chapter 3 (i.e. 8.10 mg/cm2 thickness). Proton spectra were measured across the GDR region from 16.8 to 25.6 MeV in 800 KeV steps at 7 angles from 45° to 135° (in 15° steps). In addition, spectra were measured for a number of energies at 90° to the beam with the target at angles of 45° and 135° to the beam so that any asymmetries in the target or alignment could be corrected for (see Section 2.8).

Only two spectrometer settings were used -10 and 15 MeV. The data taken with the lower energy setting (10 MeV) were normalized to the higher energy setting data (15 MeV) using the yield ratio in the overlapping energy region of the thick Nickel spectra measured at 29 MeV on both settings. These overlapping factors varied little
with angle \((1.1 \pm 0.05)\) for the ratio of yields in the 15 MeV/10 MeV spectra) indicating a degree of consistency in this method.

No effort was made to separate protons and other charged particles (mainly alphas) using thin Aluminium foils (see Section 2.4.3). However, since the chief intention of this experiment was the derivation of \(^{27}\text{Al}(\gamma,p_{0})\) and \(^{27}\text{Al}(\gamma,p_{1})\) cross sections at each angle, the presence of alpha particles in the spectra should have little effect since few alpha particles appear in the relevant energy regions due to the differences in thresholds and nuclear recoil (see Section 3.2).

After making the necessary corrections to the raw proton spectra, ground state photoproton cross sections for \(^{27}\text{Al}\) were derived at 7 different angles using the procedure described in Section 3.3.1. The top 400 KeV of each proton spectrum was discarded from the analysis to minimize the effect of the uncertainty in the virtual photon spectrum in the tip region. The ground state cross sections so derived are displayed in Figure 4.1.

![Graph showing differential cross sections for \(^{27}\text{Al}(\gamma,p)\) at 7 angles.](image)

**Fig. 4.1** \(^{27}\text{Al}(\gamma,p_{0})\) at 7 Angles

The first excited state cross sections were then deduced by subtracting the ground
state contribution from each proton spectrum in the manner described in Section 3.3.2. However the separation between the second and third excited states of $^{26}$Mg is only 1.1 MeV and the spectra were measured in 800 KeV intervals. Thus only the top 200 KeV of the truncated spectra was ignored rather than the top 400 KeV as for the ground state cross section so that the $^{27}$Al($\gamma$,p1) cross section points could be determined over the whole energy range. The first excited cross sections so derived are displayed in Figure 4.2.

![Differential Cross Section vs Photon Energy](image)

It can be readily seen that there is much interesting structure in both the $^{27}$Al($\gamma$,p0) and $^{27}$Al($\gamma$,p1) cross sections at each angle measured. Clearly these photonuclear reactions are neither isotropic in their behaviour nor are they symmetric around 90°. Thus an entirely statistical reaction process is not valid.

The total strength, integrated over energy, of the decays to the ground and first excited states as well as their ratio is plotted as a function of angle in Figure 4.3. A
compound nuclear reaction process should be symmetrical about 90° (Ho71), however, both these reactions are strongly forward peaked indicating the presence of direct reaction effects or multipole interference. In a direct reaction the incident momentum direction is important because there are few nucleon-nucleon interactions: the direction of emitted particles is highly correlated with the beam direction.

Since the spectra were measured only in 800 KeV steps, cross sections to higher excited states could not be extracted from the data using the technique described in Section 3.3.4. Only by assuming mean values for the energies of the groups of populated residual states (as done in Section 6.3.3) could meaningful cross sections be derived. However, the angular distribution analysis and formalism requires complete knowledge of the reaction channel, including the final state spin, so that little purpose would be achieved in deriving these cross sections.

### 4.3 Analysis of Data

$^{27}$Al has a ground state spin of $5/2^+$. By virtue of the multipolarity selection rules $E1$ excitation leads to intermediate states of spin $\{3/2^-, 5/2^-, 7/2^-\}$, while $E2$ excitation leads to intermediates of spin $\{1/2^+, 3/2^+, 5/2^+, 7/2^+, 9/2^+\}$. Experimental evidence
that quadrupole excitation might be significant in the GDR region of $^{27}\text{Al}$ was seen in the inelastic alpha particle scattering experiment performed by Youngblood et al. (Y077). An isoscalar quadrupole resonance centred at about 20 MeV with a width greater than 7 MeV - similar in shape and location to the E1 GDR - was observed. On the other hand M1 strength in $^{27}\text{Al}$ is expected at $35A^{-1/3}$ (Ha77) which is in the vicinity of 12 MeV and well below the E1 giant resonance region. Higher order multipoles, such as E3 and M2, should not be significant since their theoretical sum rule strengths are negligibly small in comparison to the E1 strength.

### 4.3.1 Expansion in Terms of Legendre Polynomials

If the photonuclear reaction is treated as a two-step process through well defined intermediate states then the differential cross section to a specific residual state can be expanded in terms of Legendre Polynomials (see Carr and Baglin, Ca71):

$$\frac{d\sigma}{d\Omega}(E, \theta) = \sum_{i=0}^{4} A_i(E) P_i(\cos \theta)$$

(4.1)

The $A_i(E)$ are energy dependent coefficients which are functions of the electromagnetic matrix elements governing the transition to the residual state in question. If only E1 and E2 multipole excitation is assumed then an expansion up to fourth order ($i = 4$ in Equation 4.1 is sufficient. In this case, the odd coefficients arise from interference between the E1 and E2 matrix elements, while the even coefficients contain the E1 and E2 matrix elements separately.

The possible decay channels to the $^{26}\text{Mg}$ ground state through both E1 and E2 intermediate states are summarized in Figure 4.4. Clearly even the ground state decay scheme is very complex because of the multiplicity of possible intermediate states. However since the residual state spin is $0^+$ only a channel spin $S = 1/2$ is possible. Hence there is a direct correlation between the multipolarity of the excited intermediate state and the angular momentum of the emitted nucleon.

In channel spin formalism the $A_0$ term of Equation 4.1 is, for the $^{27}\text{Al}(\gamma,p0)$ reaction:

(all E2 terms)

$$A_0 = 4p_{3/2}^2 + 6f_{5/2}^2 + 8f_{7/2}^2$$

(all E1 terms)

$$+ 2s_{1/2}^2 + 4d_{3/2}^2 + 6d_{5/2}^2 + 8g_{7/2}^2 + 10g_{9/2}^2$$

(4.2)

(all E2 terms)
In this notation, $p_{3/2}$, for example, is the matrix element through which the $3/2^-$ intermediate state is excited by E1 absorption and then decays to the ground state by emission of a $p_{3/2}$ proton.

For the decay to the first excited state ($2^+$) there are two possible channel spins: $S=3/2$ and $S=5/2$ and the theoretical description of the $A_i$ coefficients is far more complicated (Ca71 and Ma74). Since the final state spin is not 0$^+$ there is no direct correlation between the spin of the resonance state and the angular momentum of the emitted proton. For example, the $3/2^-$, $5/2^-$, $7/2^-$ E1 states of $^{27}$Al can decay by emitting particles of various spin to the first excited state:

$3/2^- \rightarrow 2^+ : p_{1/2}, p_{3/2}, f_{5/2}, f_{7/2}$

$5/2^- \rightarrow 2^+ : p_{1/2}, p_{3/2}, f_{5/2}, f_{7/2}$

$7/2^- \rightarrow 2^+ : p_{3/2}, f_{5/2}, f_{7/2}$

Because of this additional complexity the first excited state cross sections will be
discussed later (in Section 4.5) and only the ground state cross sections will be considered in the next sections.

4.3.2 $A_i$ Coefficients for the Ground State Cross Section

From the previous discussion it should be appreciated that if the reaction proceeds only through El intermediate states, the differential cross section will be symmetric about 90° since the coefficients of the asymmetric terms ($P_1$ and $P_3$) will not appear. $A_4$ will also be zero since it contains E2 matrix elements exclusively. However, there is marked angular asymmetry in the ground state cross section (see Figs. 4.1 and 4.3) so expansion up to fourth order is required.

Accordingly the ground state cross section data were summed in both 200 and 400 KeV energy bins and a Legendre-Polynomial series was fitted at each mean energy. The $A_i$ coefficients plotted as a function of energy are displayed in Figure 4.5. The $A_2/A_0$, $A_2/A_0$ coefficients are compared with $A_0$, which represents the total cross section for the $^{27}$Al($\gamma$,p$_0$) reaction, in Figure 4.6 (note that elsewhere in this chapter the notation $a_1 = A_1/A_0$, $a_2 = A_2/A_0$ is occasionally used).
It can be readily seen that there is considerable E1-E2 interference across the giant
resonance region because of the non-zero value of $A_1$. The general trends in
behaviour of these coefficients to be noted are:

(i) $A_1/A_0$ is generally positive, increasing slowly with energy.

(ii) $A_2/A_0$ is initially positive and decreases slowly to a value of about -0.4.

(iii) $A_3, A_4$ fluctuate about zero with very large uncertainties.

Defining the error weighted mean value as:

$$
\bar{A} = \frac{\sum_{i=1}^{k} A(i)/\delta A(i)^2}{\sum_{i=1}^{k} 1/\delta A(i)^2}
$$

(4.2)

with fluctuation:

$$
\delta A = \frac{1}{\sqrt{k}} \left[ \sum_{i=1}^{k} (A(i) - \bar{A})^2 \right]^{1/2}
$$

(4.3)

and then calculating error weighted mean values for the $a_1$ and $a_2$ coefficients over 3
representative regions of the ground state cross section gives:

*Table 4.1: $A_i$ Coefficient values over Representative Regions*

<table>
<thead>
<tr>
<th>Energy Region (MeV)</th>
<th>$A_1/A_0$</th>
<th>$A_2/A_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I 15.0 - 17.4</td>
<td>0.054 ± 0.24</td>
<td>0.347 ± 0.280</td>
</tr>
<tr>
<td>II 17.6 - 21.0</td>
<td>0.185 ± 0.119</td>
<td>-0.197 ± 0.178</td>
</tr>
<tr>
<td>III 21.2 - 25.0</td>
<td>0.287 ± 0.212</td>
<td>-0.384 ± 0.435</td>
</tr>
</tbody>
</table>

The dominant configuration of the $^{27}$Al ground state has 11 nucleons in the 1d$_{5/2}$
subshell. Even if there are admixtures into this ground state of more complicated
configurations, as discussed by Wildenthal et al (Wi 73), the E1 transitions from the
1d$_{5/2}$ subshell will still far outweigh E1 transitions from the 2s$_{1/2}$ and 1d$_{3/2}$ subshells
because of the far greater number of nucleons available for such transitions. The
1d$_{5/2}$ nucleons can be excited into the 1f$_{7/2}$, 2p$_{3/2}$, or 1f$_{5/2}$ orbitals according to the E1
selection rules. Excitation from the deeper 1p shell can also occur but this will lead to
negative parity residual states and not the ground state.

The relative strengths and angular distributions of the various excitations which can
contribute to the ground state cross sections are listed in Table 4.2. The assumption is
made that only these transitions will lead to formation of the ground state.

Table 4.2: Single Particle Transitions

<table>
<thead>
<tr>
<th>Transition</th>
<th>Relative Strength</th>
<th>Angular Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1d&lt;sub&gt;5/2&lt;/sub&gt;→1f&lt;sub&gt;7/2&lt;/sub&gt;</td>
<td>80%</td>
<td>1-0.35P&lt;sub&gt;2&lt;/sub&gt;</td>
</tr>
<tr>
<td>1d&lt;sub&gt;5/2&lt;/sub&gt;→2p&lt;sub&gt;3/2&lt;/sub&gt;</td>
<td>16%</td>
<td>1-0.10P&lt;sub&gt;2&lt;/sub&gt;</td>
</tr>
<tr>
<td>1d&lt;sub&gt;5/2&lt;/sub&gt;→1f&lt;sub&gt;5/2&lt;/sub&gt;</td>
<td>4%</td>
<td>1+0.45P&lt;sub&gt;2&lt;/sub&gt;</td>
</tr>
</tbody>
</table>

The El matrix elements should dominate the A<sub>2</sub>/A<sub>0</sub> terms listed in Table 4.1 since the E2 strength is expected to be only a few percent of the El strength. Simplistically then the particle-hole configurations in Table 4.2 can be associated with the regions of strength in Table 4.1. In region I the 1d<sub>5/2</sub>→1f<sub>5/2</sub> transition appears to dominate because A<sub>2</sub>/A<sub>0</sub> is positive. In region II, A<sub>2</sub>/A<sub>0</sub> is negative with a value intermediate between the theoretical values for 1d<sub>5/2</sub>→2p<sub>3/2</sub> and 1d<sub>5/2</sub>→1f<sub>7/2</sub>. In region III, the relatively large negative value of A<sub>2</sub>/A<sub>0</sub>, despite the correspondingly large errors, may be associated with the 1d<sub>5/2</sub>→1f<sub>7/2</sub> transition.

4.3.3 El Configurations in the 27Al(γ,p<sub>0</sub>) Cross Section

This correlation of the A<sub>2</sub>/A<sub>0</sub> coefficient ratio with the lp-lh configurations is too simplistic. It has been assumed that only one configuration contributes in each region whereas there are more likely to be several which can strongly interfere. For example, if both p<sub>3/2</sub> and f<sub>7/2</sub> terms contribute then A<sub>2</sub>/A<sub>0</sub> can be written as:

\[
A_2/A_0 = -\frac{0.4p^2 + 8.113pf \cos(p,f) - 2.857f^2}{4p^2 + 8f^2}
\]  (4.5)

Where p = p<sub>3/2</sub> and f = f<sub>7/2</sub>

A<sub>2</sub>/A<sub>0</sub> can now be positive if the 8.113 cos(p,f) term is greater than the other terms in the numerator. This is a more likely explanation for the positive value of A<sub>2</sub>/A<sub>0</sub> than the d<sub>5/2</sub>→f<sub>7/2</sub> spin-flip transition which should be weak. In any case, f-wave emission in the low energy region will be inhibited by the angular momentum barrier.

Furthermore, in contrast to the idea of configurations changing through the GDR is the near constancy of the angular distributions noted by other workers (Al64, Fr69). Because of this it has been suggested that a single configuration is dominant over the GDR and its strength is spread over many actual nuclear levels. Thus it is likely that the 27Al(γ,p<sub>0</sub>) reaction is dominated by the 1d<sub>5/2</sub>→1f<sub>7/2</sub> transition as the largeness of
the unperturbed matrix element seems to suggest.

If the small (by comparison) E2 terms are neglected and the spin-flip contribution is ignored then the $A_2/A_0$ ratio is given exactly by Equation 4.5. Also if the phase differences between the p, f waves are assumed to be given using R-matrix theory as described in Appendix B then the relative strengths of these two terms can be determined by solving this quadratic equation for the ratio $p/f$. The ratios $p/f$ and $8f^2/(4p^2+8f^2)$, which is the fraction of f-wave to p-wave strength in the ground state channel (see Table 4.3), are listed in Table 4.3 in 800 KeV intervals. In cases where a complex solution was obtained for the ratio $p/f$, the real part was taken.

It is clear from Table 4.3 that in general the $f_7/2$ contribution dominates the $^{27}$Al($\gamma$,p)$_0$ reaction although there is significant $p_{3/2}$ contribution as well. In the main part of the GDR (see Figure 4.6), from about 20 to 22 MeV, the decay is almost entirely by $f_7/2$ emission. Below this region, in the peaks at about 175 and 18.6 MeV, there is considerable by $p_{3/2}$ emission accounting for the positive value of $A_2/A_0$.

Allowing for the approximations made, the earlier suggestion that one configuration dominates in the GDR (or at least the fractions that decays to the ground state) appears to be justified.

Table 4.3: Contribution of p and f-wave terms to the $^{27}$Al($\gamma$,p)$_0$ Reaction

<table>
<thead>
<tr>
<th>Energy</th>
<th>Ratio $p/f^1$</th>
<th>$8f^2/(4p^2+8f^2)$ (as percentage)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.0</td>
<td>0.29</td>
<td>96*</td>
</tr>
<tr>
<td>16.8</td>
<td>0.57</td>
<td>86</td>
</tr>
<tr>
<td>17.6</td>
<td>0.95</td>
<td>69</td>
</tr>
<tr>
<td>18.4</td>
<td>0.78</td>
<td>77</td>
</tr>
<tr>
<td>19.2</td>
<td>0.62</td>
<td>84</td>
</tr>
<tr>
<td>20.0</td>
<td>0.39</td>
<td>93</td>
</tr>
<tr>
<td>20.8</td>
<td>0.30</td>
<td>96</td>
</tr>
<tr>
<td>21.6</td>
<td>0.09</td>
<td>100</td>
</tr>
<tr>
<td>22.4</td>
<td>0.87</td>
<td>72*</td>
</tr>
<tr>
<td>23.2</td>
<td>0.16</td>
<td>99</td>
</tr>
<tr>
<td>24.0</td>
<td>0.80</td>
<td>76*</td>
</tr>
<tr>
<td>24.8</td>
<td>0.46</td>
<td>90</td>
</tr>
</tbody>
</table>

$^1$ Imaginary solution for the ratio $p/f$ was obtained and only the real part taken.
Imaginary solution for the ratio $p/f$ was obtained and only the real part taken

4.3.4 Previous Work on Neighbouring Nuclei

At this stage it is worthwhile considering the study of the angular distributions of ground state protons in the neighbouring nuclei $^{26}\text{Mg}$ and $^{28}\text{Si}$ to see if similar trends are noticed.

The $^{27}\text{Al}(p,\gamma_0)$ reaction has been studied by Singh et al (Si). These authors observed that the angular distributions vary little with energy and do not appear to be closely correlated with the giant resonance structure. Two possible solutions for the $^{27}\text{Al}(p,\gamma_0)$ reaction were obtained from the analysis: one solution had $2p_{3/2}$ proton capture into the GDR dominant; the other had $1f_{7/2}$ proton capture predominant. The solution with $2p_{3/2}$ proton capture was slightly favoured because the anomalous gamma ray angular distribution is thus better explained and the $a_2$ value is always

![Graph](image-url)
closer to that expected for pure $p_{3/2}$ capture than pure $f_{7/2}$ capture.

Wienhard et al (Wi77) measured the angular distributions of protons from $^{26}$Mg from 16 to 23 MeV. From the conclusions of their previously measured de-excitation spectra from $^{26}$Mg (Ba76) they assumed that up to 23.5 MeV over 80% of the $^{26}$Mg$(\gamma,p)$ reaction goes either to the ground state ($5/2^+$) in $^{25}$Na or the first excited state ($3/2^+$) at 90 keV. The non-zero $A_1$ values indicate the presence of E1-E2 interference in this energy region, but the E2 strength should be small since the $A_3$ and $A_4$ coefficients are both consistent with zero within the experimental errors. By neglecting the spin-flip $d_{5/2} \rightarrow f_{5/2}$ contribution, the authors deduce that the GDR of $^{26}$Mg decays predominantly by $2p_{3/2}$ proton emission, with a small amount of $f_{7/2}$ emission which increases with excitation energy. They concluded that there is no significant difference in the particle-hole compositions of the two main peaks observed in the $^{26}$Mg$(\gamma,p)$ cross section.

In contrast to the present results for the reaction, where $f_{7/2}$ appears to dominate, in both these nuclei $p_{3/2}$ emission (or capture in the case of the $^{27}$Al$(p,\gamma)^{28}$Si) is favoured. The most likely explanation for this disagreement is the threshold difference. The $(\gamma,p)$ thresholds for $^{28}$Si, $^{27}$Al, and $^{26}$Mg are 11.59, 8.27, 14.14 MeV respectively. The low threshold for the $^{27}$Al$(\gamma,p)$ reaction allows $f_{7/2}$ protons to be emitted even from the low energy region of the GDR whereas the angular momentum barrier and much higher thresholds will severely inhibit $f$-wave contribution to the photoproton ground state decay channels of $^{28}$Si and $^{26}$Mg.

In a later communication, Barker (Ba80) claims that Singh et al have assumed that the phase differences between the various matrix elements involved in the $^{27}$Al$(p,\gamma)^{28}$Si reaction are zero. Using non-zero phases he finds that the solution with $f_{7/2}$ dominant is actually favoured over the $p_{3/2}$ dominant solution, in contradiction to their conclusions. Thus there some reconciliation as to the apparent difference in the ground state photoproton reactions of $^{27}$Al and $^{28}$Si. Both may well occur via the same reaction mechanism, irrespective of the differences in the proton thresholds.

### 4.4 Quadrupole Strength in the Ground State Cross Section

It has been demonstrated that the $^{27}$Al$(\gamma,p\gamma)$ reaction is most likely dominated by $1d_{5/2} \rightarrow 1f_{7/2}$ transitions with a weaker $1d_{5/2} \rightarrow 2p_{3/2}$ component. However, more
information can be extracted from the behaviour of the $A_1$, $A_3$, and $A_4$ coefficients which arise from the E2 contribution to the $^{27}\text{Al}(\gamma,p0)$ reaction.

For the $^{16}\text{O}(\gamma,p0)$ reaction (see Ha72, Ha74) a complete solution of the various elements and phases is possible by considering both polarized and unpolarised $(p,\gamma0)$ angular distributions. The E2 strength was clearly extracted from the $A_i$ coefficients and a cross section for the E2 component of the ground state decay channel was derived.

In contrast the situation for the $^{27}\text{Al}(\gamma,p0)$ reaction is far more complicated. There are 8 unknown matrix elements (5 from E2 and 3 from E1 – see Figure 4.4) so that an exact solution cannot be obtained since there are only 5 equations relating the $A_i$ coefficients to these matrix elements (given implicitly in the tables of Carr and Baglin, Ca71). It is thus necessary to make assumptions as to which should be the strongest transitions. For the E1 component, the strongest transitions are $1d_{5/2} \rightarrow 1f_{7/2}$ and $1d_{5/2} \rightarrow 2p_{3/2}$. However, it is also necessary to consider which are the strongest E2 transitions.

### 4.4.1 The Strongest E2 Matrix Elements

The single particle matrix elements may be evaluated using the formula for multipole transitions given, for example, by Gillet and Vinh-Mau (Gi64).

\[
\langle p \mid \xi_j \mid h \rangle = \int_p \int_h \int_p \int_h \left( \begin{array}{ccc} l_p & l_p \\ 0 & 0 \\ 0 & 0 \end{array} \right) \times \frac{1}{\sqrt{2l_p+1}} \times \left( \begin{array}{ccc} l_p & 1/2 \\ j_h & 1/2 \end{array} \right) f r^2 dr u_p u_h r^4
\] (4.6)

Where $p$, $h$ represent the particle and hole wavefunctions respectively and $J$ is the spin of the excited multipole.

For E2 transitions this expression reduces to:

\[
\langle p \mid \xi_j \mid h \rangle = \frac{1}{\sqrt{10}} \int_h \int_p \frac{2}{2} \times \left( \begin{array}{ccc} j_h & 2j_p \\ 0 & 1/2 \end{array} \right) f r^4 R_1 R_2 dr
\] (4.7)

The possible E2 transitions from the $1d_{5/2}$ subshell which contribute to the ground state cross section with their strengths as calculated in Equation 4.5 are listed in Table 4.4. Excitations from the $1p$ shell are also possible but will not result in the population of positive parity residual states.

### Table 4.4: Single Particle E2 Excitations and Penetration Factors for $^{27}\text{Al}$

<table>
<thead>
<tr>
<th>Excitation</th>
<th>Clebsch Gordon x Radial Integral</th>
<th>(Matrix Element)$^2$</th>
</tr>
</thead>
</table>

13
It is immediately evident that the $1d_{5/2} \rightarrow 1g_{7/2}$ excitation is by far the strongest with 79% of the total E2 strength in the zeroth order calculation. However g-wave particles face a large angular momentum barrier as they leave the nucleus. The penetrabilities for different $l$-values and energies may be estimated from Marion and Young’s compilation of nuclear reaction data (Ma68) and some representative examples are listed in Table 4.5. Even with the reduced penetration probability $g_{9/2}$ emission should dominate the E2 component of the $^{27}$Al ($\gamma$,p$_0$) reaction, particularly above 20 MeV.

<table>
<thead>
<tr>
<th>$l$-Value</th>
<th>Energies 16</th>
<th>Energies 20</th>
<th>Energies 24</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l = 0$</td>
<td>2.5</td>
<td>3.5</td>
<td>4.5</td>
</tr>
<tr>
<td>$l = 1$</td>
<td>2.3</td>
<td>2.5</td>
<td>4.5</td>
</tr>
<tr>
<td>$l = 2$</td>
<td>1.7</td>
<td>2.5</td>
<td>4.0</td>
</tr>
<tr>
<td>$l = 3$</td>
<td>0.9</td>
<td>2.0</td>
<td>3.5</td>
</tr>
<tr>
<td>$l = 4$</td>
<td>0.3</td>
<td>1.3</td>
<td>2.5</td>
</tr>
</tbody>
</table>

### 4.4.2 The Simplified Coefficients

From the last few sections it is clear that the strongest E1 components of the GDR are $f_{7/2}$ and $p_{3/2}$ while the strongest E2 component is $g_{9/2}$. If all other matrix elements are neglected, the $A_i$ coefficients which describe the $^{27}$Al ($\gamma$,p$_0$) reaction may be simplified considerably.

\[
A_0 = 8f^2 + 4p^2 \tag{4.8a}
\]
\[
A_1 = 21.909fg\cos(f, g) \tag{4.8b}
\]
\[
A_2 = -2.857f^2 - 0.4p^2 + 8.113pf\cos(p, f) \tag{4.8c}
\]
\[
A_3 = -6.260fg\cos(f, g) + 12.344pg\cos(pg) \tag{4.8d}
\]
\[
A_4 = -2.857g^2 \tag{4.8e}
\]

Where $g = g_{9/2}$, $f = f_{7/2}$, $p = p_{3/2}$ as before.

Since the $A_4/A_0$ coefficient tends to correspondingly decrease (refer to Figure 4.5), this
suggests that the amount of f\(7/2\) and g\(9/2\) emission increases with energy. Furthermore the \(A_3\) coefficient should be dominated by the first term (which has the opposite phase to \(A_1\)) and decrease with energy, in agreement with observations.

Little can be said about the \(A_4/A_0\) ratio, except that it fluctuates about zero. This suggests that the E2 strength in the \(^{27}\text{Al}(\gamma,p_0)\) reaction is small since \(A_4\) contains only E2 terms.

### 4.4.3 Solution of the Quadrupole Terms

Because of the large fluctuations on the \(a_3\) and \(a_4\) values, these cannot be sensibly used to deduce the g-wave amplitudes. However the relative strengths of the f\(7/2\) and g\(9/2\) matrix elements can be found from the \(A_1\) coefficient, simplified as in Equation 4.8:

\[
g = \frac{A_1}{21.909 \cos(f,g)}
\]

Having found the value of \(f\) at each energy and assuming the phases between the f and g-waves to be given by the method described in Appendix B, the strength of the g\(9/2\) matrix element can then be deduced from this equation. The contribution of the g\(9/2\) matrix element to the \(^{27}\text{Al}(\gamma,p_0)\) reaction is \(10(g_{9/2})^2\) from Equation 4.2 and this is effectively the E2 component of the ground state cross section:

\[
\sigma_0(E2) = 10g^2
\]

Figure 4.7 shows the E2 cross section derived in this fashion with the total cross section (represented by \(A_0\)) for comparison. The E2 cross section shows some resonant structure peaking at about 22 MeV with a value of 2\(\mu\)b/sr. The width is 7-8 MeV, roughly in agreement with the E2 giant resonance observed in inelastic alpha particle scattering by Youngblood et al (Yo77).

This E2 strength is isoscalar (T=0) rather than isovector (T=1). According to Bohr and Mottelson (Bo76), the T=0 and T=1 components of the quadrupole strength are situated at about \(60A^{-1/3}\), and \(135A^{-1/3}\) MeV respectively. Thus, for \(^{27}\text{Al}\), the T=0 resonance should be located at 20 MeV while the T=1 resonance should be located at 45 MeV, far too high to cause appreciable interference with the E1 giant resonance. Monopole strength, which has not been considered so far, should be located at \(60A^{-2/3}\) and \(178A^{-1/3}\) for T=0 and T=1 respectively (Su73). However the monopole mode
cannot be excited by photon interactions and so cannot have any effect on the $^{27}\text{Al}(\gamma,p_0)$ cross section.

The isoscalar E2 sum rule (Ge53) gives a maximum absorption cross section of:

$$K(L)_{LM}!J = \frac{\sigma(E2)}{E^2} \, dE = \frac{0.22z^2}{A^{1/3}} \, \mu b/MeV$$

(4.11)

Which is 12.4\(\mu\)b/MeV for $^{27}\text{Al}$.

The total integrated E2 strength in the $^{27}\text{Al}(\gamma,p_0)$ reaction, estimated from Figure 4.7 is 0.31±0.10 \(\mu\)b/MeV. Hence the E2 component of the ground state cross section exhausts only about 2.5\% of the sum rule. The E2 strength observed in the ground state proton decay of the neighbouring nuclei $^{26}\text{Mg}$, $^{28}\text{Si}$ (Wi77, Si65) is also weak, in qualitative agreement with the present result.

The E2 strength is at most a few percent of the total $^{27}\text{Al}(\gamma,p_0)$ cross section (Figure 4.7) favouring the interpretation that the fraction proceeds almost entirely via the
GDR with only a small contribution from the GQR (Giant Quadrupole Resonance).

4.5 The Angular Distribution of the $^{27}$Al($\gamma$,p$_1$) Reaction

Series of Legendre Polynomials were also fitted to the first excited state cross section data. Figure 4.8 shows the $a_1$, $a_2$ coefficients plotted with the $A_0$ coefficient for comparison. The general behaviour of the $a_1$ and $a_2$ coefficients for the reaction is similar to that noticed in the $^{27}$Al($\gamma$,p$_0$) reaction (see Section 4.3.2):

i. $A_1/A_0$ increases slowly with energy
ii. $A_2/A_0$ decreases with energy.

Interpretation of these data is far more complex than for the angular distribution. The $^{27}$Al($\gamma$,p$_1$) reaction may proceed via two possible channel spins: $S=3/2$ and $S=5/2$ and from a multiplicity of intermediate E1 and E2 states. Unlike the ground state reaction
there is no direct correlation between the spin of the intermediate state and the angular momentum of the emitted particle. For example, the $7/2^-$ excited state can decay via either $L=1$ (p) or $L=3$ (f) proton emission to the first excited state of $^{26}\text{Mg}$ ($2^+$).

Similar complications occurred in the analysis of the $^{27}\text{Al}(\gamma,p_1)$ data of Singh et al (Si65). For dipole gamma radiation the $^{27}\text{Al}(\gamma,p_1)$ reaction can only proceed through $1^-$ intermediate states since the ground state of $^{28}\text{Si}$ is $0^+$. However the $^{27}\text{Al}(\gamma,p_1)$ reaction may proceed through $1^-, 2^-, 3^-$ intermediate states to the first excited state by dipole radiation. Because of the nearly invariant angular distribution of this reaction the authors assumed that a single configuration dominates the giant resonance and compared the range of theoretical values for each intermediate state ($1^-, 2^-, 3^-$) with the experimental values of $a_2$. They deduced that only a spin assignment of $2^-$ was consistent with the data.

Similarly to interpret the $^{27}\text{Al}(\gamma,p_1)$ angular distribution data in a meaningful fashion it is necessary to assume that a single configuration dominates the giant resonance. Under these conditions the possible values of $a_2$ taken from the tables of Carr and Baglin (Ca71) are displayed in Table 4.6. If the $^{27}\text{Al}(\gamma,p_1)$ reaction proceeds predominantly through 1p-1h configurations and the excited particle is emitted, there should be little $f_{5/2}$ proton emission since the $1d_{5/2} \rightarrow 1f_{5/2}$ spin-flip transition is relatively weak.

*Table 4.6: Values of $a_2$ Coefficient for the $^{27}\text{Al}(\gamma,p_1)$ Reaction assuming transition through various single intermediate resonances.*

<table>
<thead>
<tr>
<th>Channel Spin</th>
<th>Emitted Particle</th>
<th>Resonance State Spin</th>
</tr>
</thead>
<tbody>
<tr>
<td>S=3/2</td>
<td>$f_{5/2}, f_{7/2}$</td>
<td>$3/2^- 5/2^- 7/2^-$</td>
</tr>
<tr>
<td></td>
<td>$p_{1/2}$</td>
<td>-0.08 0.25 -0.24</td>
</tr>
<tr>
<td>S=5/2</td>
<td>$f_{5/2}, f_{7/2}$</td>
<td>0.02 -0.02 -0.07</td>
</tr>
<tr>
<td></td>
<td>$p_{1/2}$</td>
<td>-0.02 -0.36 -0.21</td>
</tr>
</tbody>
</table>

*It is assumed that there is no $p_{1/2}$ emission from the giant resonance.*

The experimental observation is that $a_2$ becomes more negative with energy with a mean value of $-0.37\pm0.30$ above 20 MeV. If a single configuration only is involved,
this behaviour could be explained by the reaction proceeding through the following
channels:

i. \(S=3/2\) channel spin, \(7/2^-\) intermediate, \(f_{7/2}\) emitted

ii. \(S=5/2\) channel spin, \(5/2^-\) intermediate, \(p_{3/2}\) emitted

iii. \(S=5/2\) channel spin, \(7/2^-\) intermediate, \(p_{3/2}\) emitted

The other possible decay channels are excluded because they give values of \(a_2\) which are too small.

Thus it seems that the \(^{27}\text{Al} (\gamma, p_1)\) reaction proceeds through E1 intermediate states of spin \(5/2^-\) or \(7/2^-\) rather than through the \(3/2^-\) state. The difference in structure between the ground and first excited state cross sections may then be understood in terms of feeding from different excited states of the GDR (refer to Section 3.5.4).

Certainly for \(^{27}\text{Al} (\gamma, p_0)\) the feeding from the \(5/2^-\) intermediate state should be small because this transition involves a spin-flip. The \(5/2^-\) states that decay to the first excited state can be formed by coupling the excited core (\(2^+\)) to either a \(p_{3/2}, f_{5/2}\) or \(f_{7/2}\) (refer to Table 3.6) and so can have different configurations from the states that decay to the ground state (\(0^+\)).

The behaviour of \(a_1\) is similar to the behaviour observed in the ground state cross section. A gradual increase is seen which does not bear much correlation with the structure in the \(^{27}\text{Al} (\gamma, p_1)\) cross section. The non-zero value of this coefficient suggests that there is also a small E2 contribution to this reaction. However to reduce the \(A_i\) coefficients to tractable complexity would require too many assumptions.

**4.6 Conclusions**

The study of the angular distributions of the photoproton decay to the ground and first excited states of \(^{26}\text{Mg}\) has revealed several interesting phenomena:

i. The anisotropy of both reactions indicates the presence of multipole interference.

ii. Using suitable assumptions it was shown that the ground state decay channel was dominated by \(f_{7/2}\) emission with some contribution from \(p_{3/2}\) emission.

iii. The E2 component of the ground state cross section was determined by assuming that the \(g_{9/2}\) matrix element was the only significant quadrupole
term. The E2 strength exhausted only 2.5% of the T=0 sum rule thus justifying earlier assumptions that the $^{27}\text{Al}(\gamma,p_0)$ reaction is principally dipole.

iv. The $A_i$ coefficients for the $^{27}\text{Al}(\gamma,p_1)$ reaction show similar trends to the $^{27}\text{Al}(\gamma,p_1)$ reaction. By assuming a single intermediate state configuration it was shown that $p_{3/2}$ emission is more probable than $f_{7/2}$ emission, although a mixture of these is likely.